# Notes and Examples for Part 5:

- Circles
- Parabolas
- Ellipse
- Hyperbola

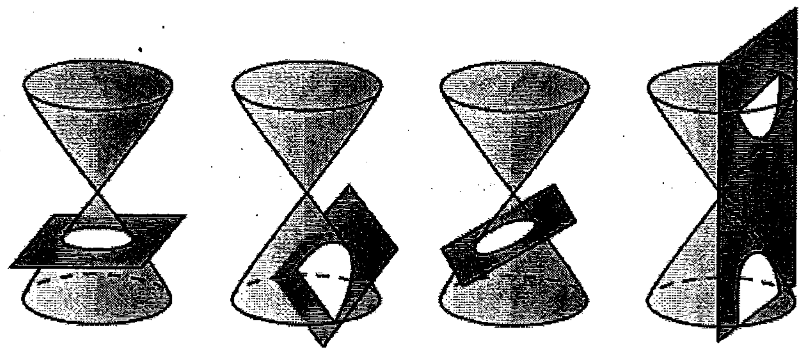
**Algebra 2 Notes** Name: ____________________

**Conic Sections Unit** Date: ________________ Pd: ____

# Conic Sections Reference Sheet

## Key Concept Conic Sections

A **conic section** is a curve you get by intersecting a plane and a double cone. By changing the inclination of the plane, you can get a circle, a parabola, an ellipse, or a hyperbola.



**Conic Sections**

| | | | |
|---|---|---|---|
| Circle | $(x-h)^2+(y-k)^2=r^2$ | center $(h,k)$<br>$r$ = radius | |
| Parabola | $y=a(x-h)^2+k$ | axis of symmetry $x=h$<br>directrix $y=k-\frac{1}{4a}$ | vertex $(h,k)$<br>focus $\left(h,\ k+\frac{1}{4a}\right)$ |
| Parabola | $x=a(y-k)^2+h$ | axis of symmetry $y=k$<br>directrix $x=h-\frac{1}{4a}$ | vertex $(h,k)$<br>focus $\left(h+\frac{1}{4a},\ k\right)$ |
| Ellipse | $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ | foci $(h\pm c,\ k)$ where $c^2=a^2-b^2$, center $(h,k)$ | |
| Ellipse | $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$ | foci $(h,\ k\pm c)$ where $c^2=a^2-b^2$, center $(h,k)$ | |
| Hyperbola | $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$ | foci $(h\pm c,\ k)$ where $c^2=a^2+b^2$, center $(h,k)$ | |
| Hyperbola | $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$ | foci $(h,\ k\pm c)$ where $c^2=a^2+b^2$, center $(h,k)$ | |

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**Unit 9: Conic Sections (Day 2)** Date: ______________ Pd: ____

## Parabolas

***Definition of Parabola:*** Set of all points in a plane that are the same distance from a given point (focus) and a given line (directrix)

| | | |
|---|---|---|
| **Form of Equation** | $y = a(x-h)^2 + k$ | $x = a(y-k)^2 + h$ |
| **Vertex** | $(h, k)$ | $(h, k)$ |
| **Axis of Symmetry** | $x = h$ | $y = k$ |
| **Focus** | $\left(h,\ k+\frac{1}{4a}\right)$ | $\left(h+\frac{1}{4a},\ k\right)$ |
| **Directrix** | $y = k - \frac{1}{4a}$ | $x = h - \frac{1}{4a}$ |
| **Direction of Opening** | Up if $a>0$<br>Down if $a<0$ | Right if $a>0$<br>Left if $a<0$ |
| **Length of Latus Rectum (LLR)** | $\left\lvert\frac{1}{a}\right\rvert$ units | $\left\lvert\frac{1}{a}\right\rvert$ units |

1. $3(y-3) = (x+6)^2$

y= equation

① solve for y

$y - 3 = \frac{1}{3}(x+6)^2$

$\boxed{y = \underset{a}{\frac{1}{3}}(x+\underset{h}{6})^2 + \underset{k}{3}}$ opens up!

② vertex: $(-6, 3)$

AOS: $x = -6$

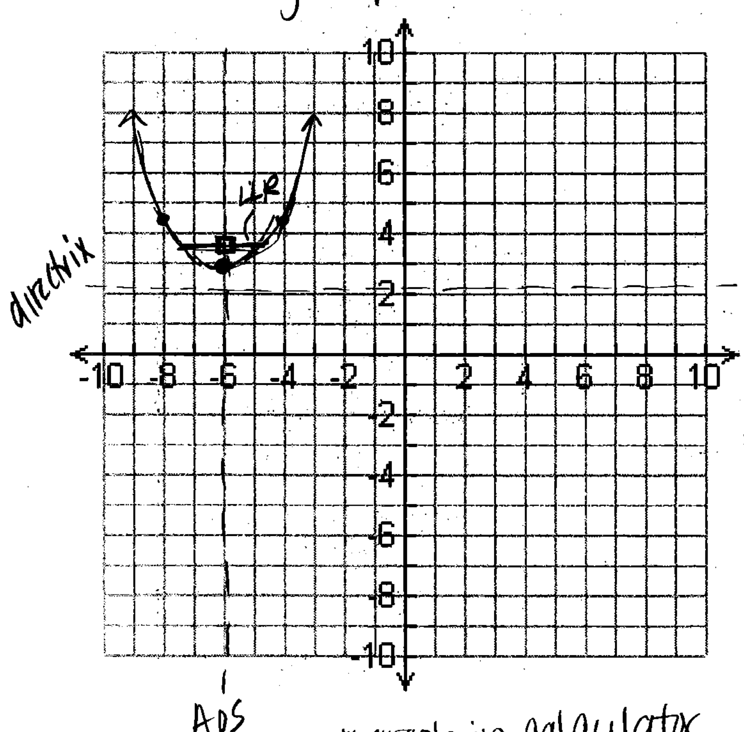
Focus: $\left(h, k + \frac{1}{4a}\right)$

$\left(-6, 3 + \frac{1}{4(1/3)}\right) = (-6, 3\,3/4)$

Directrix: $y = k - \frac{1}{4a} = 3 - \frac{1}{4(1/3)}$

$y = 2\frac{1}{4}$

LLR: $\left\lvert\frac{1}{1/3}\right\rvert = 3$




* graph in calculator to check!! *

x = opens horizontally

2. $x=\overset{a}{\frac{1}{4}}(y+\overset{k}{2})^2+\overset{h}{3}$

Vertex: $(3,-2)$

AOS: $y=-2$

Focus: $\left(h+\frac{1}{4a}, k\right)$

$\left(3+\frac{1}{4(1/4)}, -2\right)$

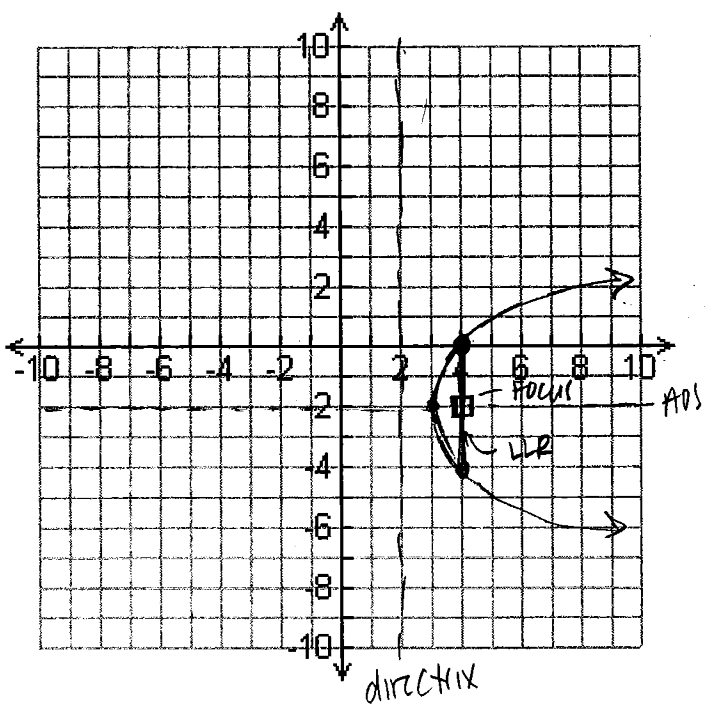
$=(3+1,-2)=(4,-2)$

Directrix: $x=h-\frac{1}{4a}=3-\frac{1}{4(1/4)}$

$x=2$

LLR: $\left|\frac{1}{1/4}\right| = 4$ units

opens right!




x = opens horizontally

complete the square!!

3. $4x=y^2+2y+13$

$4x-13+\boxed{1}=y^2+2y+\boxed{1}$

$4x-12=(y+1)(y+1)$

$4x-12=(y+1)^2$

$\frac{4x}{4}=\frac{(y+1)^2+12}{4}$

$$\boxed{x=\underset{a}{\frac{1}{4}}(y+\underset{k}{1})^2+\underset{h}{3}}$$

Vertex: $(3,-1)$

AOS: $y=-1$

Focus: $\left(h+\frac{1}{4a}, k\right)$

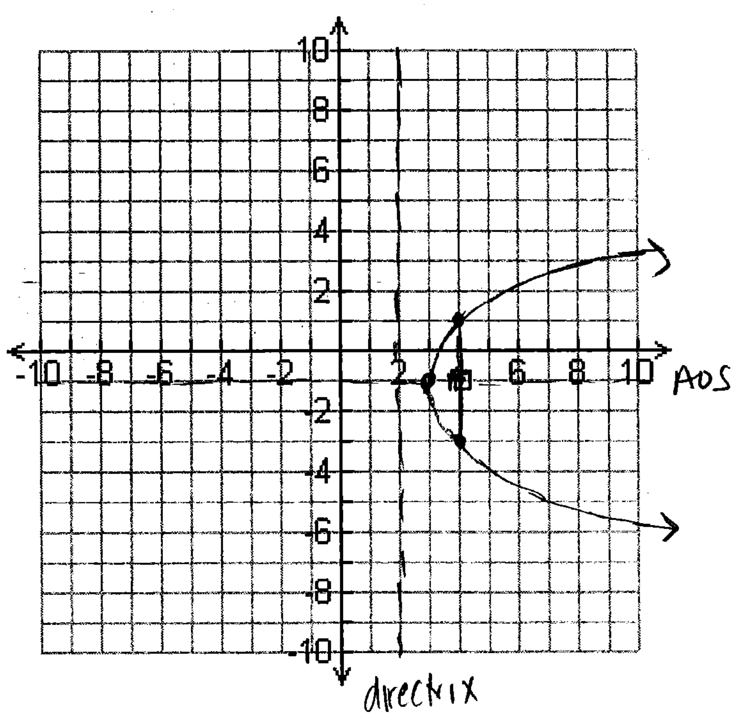
$\left(3+\frac{1}{4(1/4)}, -1\right)=(3+1,-1)$

$(4,-1)$

Directrix: $x=h-\frac{1}{4a}$

$x=3-1 \quad x=2$

LLR $=\left|\frac{1}{1/4}\right| = 4$ units



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Write an equation for the each parabola described below. Then draw the graph.

4. Vertex $(\overset{h}{5}, \overset{k}{-1})$; focus (3, -1)

① plot the points

② opens <u>left</u> $(x = \_)$

③ focus $= (h + \frac{1}{4a}, k)$ need $\underline{a}$

$= (5 + \frac{1}{4a}, -1)$

$3 = 5 + \frac{1}{4a}$

$-5 \quad -5$

$4a \cdot -2 = \frac{1}{4a} \cdot 4a$

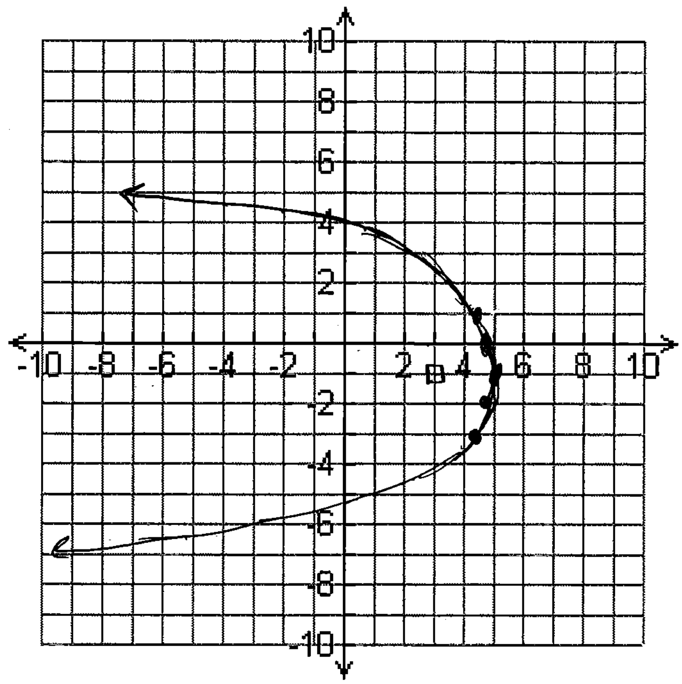
$-8a = 1$

$\boxed{a = -\frac{1}{8}}$

④ equation: $\boxed{x = -\frac{1}{8}(y+1)^2 + 5}$

⑥

| x | y |
|---|---|
| 5 | -1 |
| 4.875 | -2 |
| 4.5 | -3 |



5. Vertex $(\overset{h}{3}, \overset{k}{-7})$, directrix $y = -7\frac{1}{16}$

① plot the point + line

② opens <u>up</u> $(y = \_)$

③ directrix: $y = k - \frac{1}{4a}$

$y = -7 - \frac{1}{4a}$

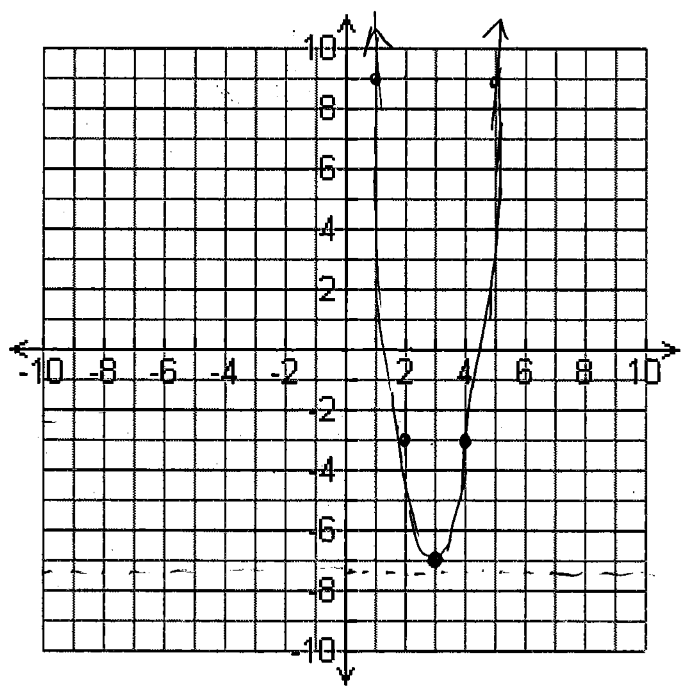
$-7\frac{1}{16} = -7 - \frac{1}{4a}$

$+7 \qquad +7$

$-\frac{1}{16} = -\frac{1}{4a}$

$\boxed{a = 4}$

④ equation: $\boxed{y = 4(x-3)^2 - 7}$



⑤

| x | y |
|---|---|
| 3 | -7 |
| 2 | -3 |
| 1 | 9 |

or graph → $\boxed{\text{TABLE}}$

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# Circles

**Definition:** A circle is the set of all points in a plane that are equidistant from a given point in the plane, called the center.

Standard form of a circle, with center $(h, k)$ and radius $r$ units is: $(x-h)^2+(y-k)^2=r^2$

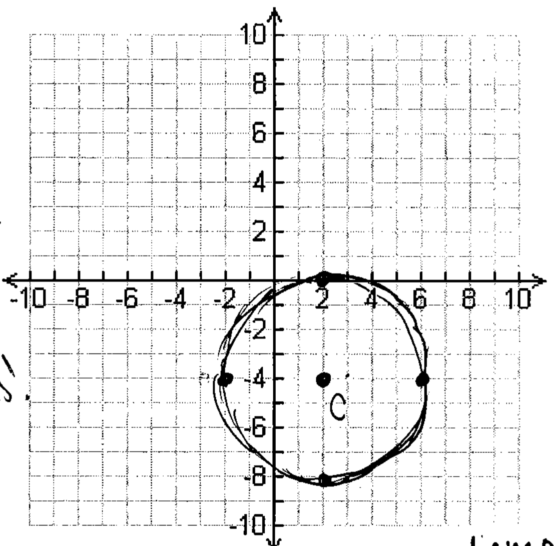
Examples

Find the center and radius of the circle with the given equations. Then graph the circle.

1. $(x-\overset{h}{2})^2+(y+\overset{k}{4})^2=16$      2. $(x-3)^2+y^2=9$

center: (3, 0)

r = 3

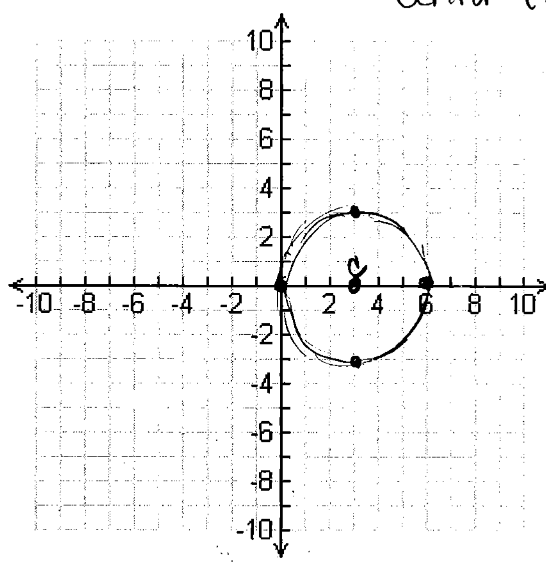



(2, -4)

r = 4

tangent to the x-axis!

no diamonds!



tangent to the y-axis!

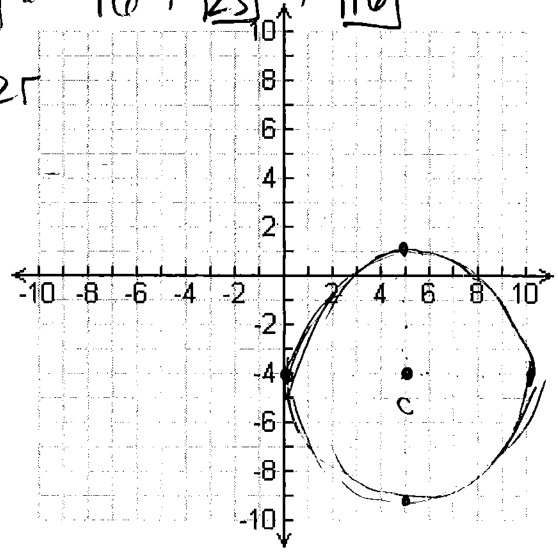
3. $x^2+y^2-10x+8y+16=0$

$x^2-10x+\boxed{(-5)^2}+y^2+8y+\boxed{(4)^2}=-16+\boxed{25}+\boxed{16}$

$(x-5)(x-5)+(y+4)(y+4)=25$

$(x-5)^2+(y+4)^2=5^2$

center (5, -4)    r = 5



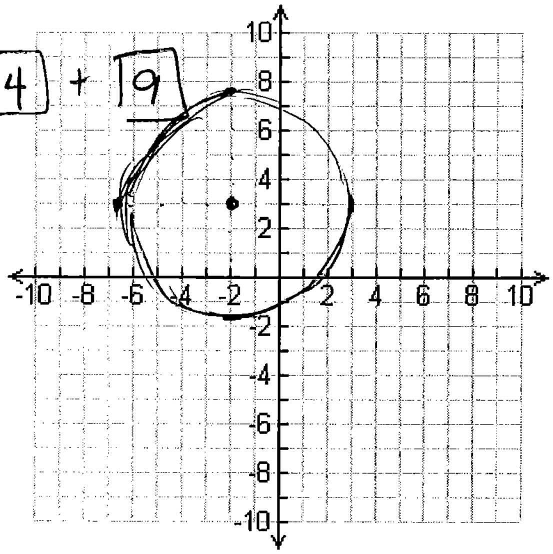
4. $x^2 + y^2 + 4x - 6y = 10$

$x^2+4x+\boxed{(2)^2}+y^2-6y+\boxed{(3)^2} = 10+\boxed{4}+\boxed{9}$

$(x+2)(x+2)+(y-3)(y-3)=23$

$(x+2)^2+(y-3)^2=23$

center $(-2,3)$ $r=\sqrt{23}$ $\approx 4.80$



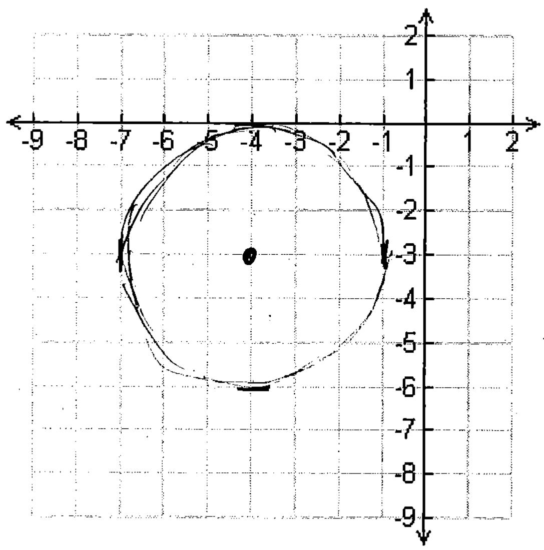
5. Write an equation for a circle with center at (–4, –3) that is <u>tangent</u> to the…

a. $x$-axis radius $=3$

$(x+4)^2+(y+3)^2=9$

b. $y$-axis radius $=4$

$(x+4)^2+(y+3)^2=16$



6. Write an equation for a circle if the endpoints of a diameter are at (2, 8) and (2, –2).

center $=(2,3)$ $r=5$

$(x-2)^2+(y-3)^2=25$